

## Physics 681-481; CS 483: Assignment #6

### Searching for one of 4 items

(please hand in after the lecture, Thursday, April 20)

We explore here Grover's algorithm when it is applied to identify one of only four items. This turns out to be a special case in two ways. (1) The probability of success after just a single query of the oracle turns out to be exactly 1. (2) The general procedure for constructing  $\mathbf{W} = 2|\phi\rangle\langle\phi| - \mathbf{1}$  out of Toffoli gates does not apply when the input register has only two qubits, and a different (simpler) construction must be used, special to the case  $|\phi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$ .

1. To convince yourself that Grover's algorithm is not entirely uninteresting when  $N = 4$ , note that if you have a classical oracle that indicates which of four items is special, all you can do is apply the oracle to one randomly selected item after another. What is the mean number of queries of the classical oracle required to determine which item is special? [Note: It is never necessary to query the oracle 4 times. (*Why?*)]

2. Figure 1 shows four possibilities for a quantum oracle  $\mathbf{U}_f$ . The top two wires carry the input register; the bottom wire is the output register, which is flipped if and only if the top two wires represent the special number. If the most significant bit is carried by the top wire (the usual convention), identify which of the four possibilities is associated with the special number being 00, 01, 10, and 11. Explain your identification in a sentence or two, by writing a caption for the figure.

3. Write a caption for Figure 2, explaining why it acts as  $\mathbf{1} - 2|11\rangle\langle 11|$ .

4. In Figure 3 the 3-qubit gate labeled ? is one of the four oracles shown in Figure 1. Write a caption for the figure that explains why this circuit, applied to the input  $|0\rangle|0\rangle|1\rangle$  produces a single Grover iteration.

5. Write a second part to the caption of Figure 3 explaining why the final state of the input register is exactly the state marked as special by the oracle. This might be simplest to explain by writing down the explicit form of the state emerging from the oracle in case (3) of Figure 1 and then using the identification of  $\mathbf{W}$  you made in the first part of the figure caption, to calculate directly what the rest of the circuit does to that state. Cases (0), (1), and (2) are then easily related to case (3) in a few more sentences.

6. Explain, directly in terms of the angles appearing in the two-dimensional geometrical analysis in Chapter 4 of the lecture notes, why a single query of the oracle succeeds in this case with probability 1.

Figure 1

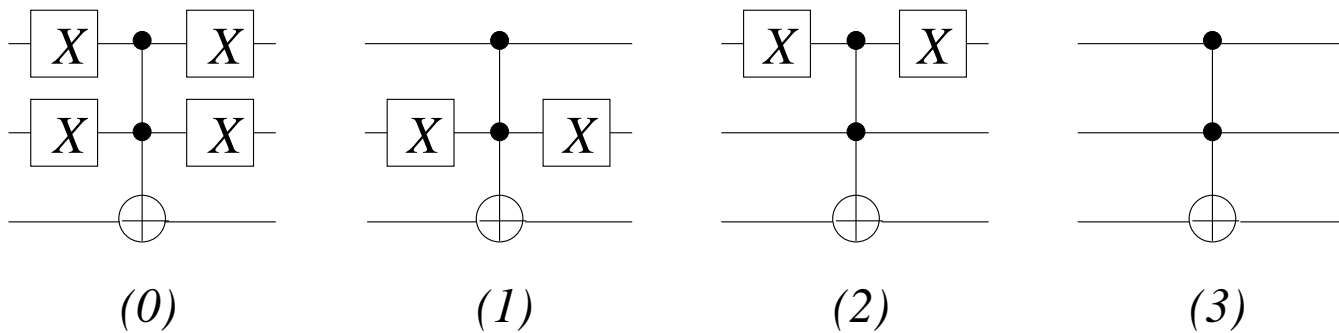


Figure 2

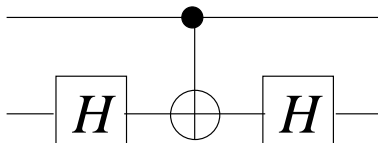


Figure 3

