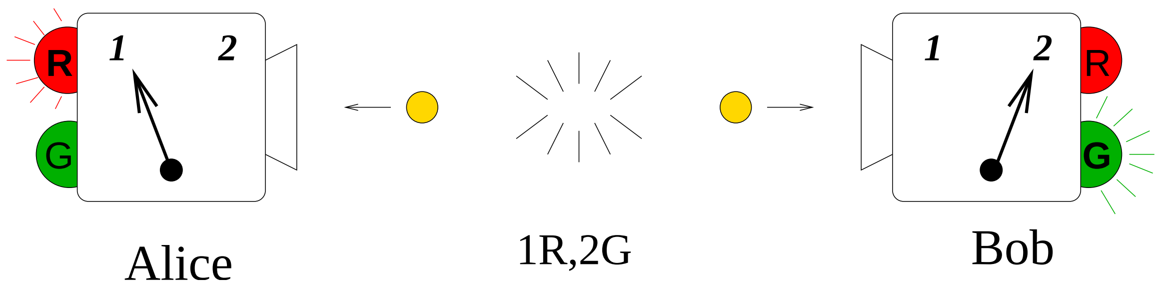
Notes for a talk to the Society of Physics Students, 3/8/05:
Spooky quantum correlations: Lucien Hardy's experiment



Each detector functions independently, in one of two modes: 1 or 2.
Detectors set to 1 or 2 by tossing coin at each, after particles leave source.
Each detector produces one of two outputs: R (red) or G (green).

Features of Data from Many Different Runs

Alice	**Bob**	**Data**
1	2	never both G
2	1	never both G
1	1	never both R
2	2	sometimes both G

Some plausible assumptions about the data:

(a) Because the detectors function independently of one another, the explanation for the above correlations lies entirely in the fact that the particles triggering the detectors originated from a common source.

(b) Because the detectors are set randomly and independently after the particles have left the source, the switch settings cannot affect any features the particles may have acquired when they were together at the source.

(c) Because each detector is triggered by a single particle, whatever features of the particle the detector responds to can reside only in that particle — not in the particle that triggered the other detector.

Analysis leading to trouble:

(1) Any run might be a 12 or 21 run. So if one particle *allows* a type 2 detector to flash G, its companion must *require* a type 1 detector to flash R. Otherwise 1G,2G and 2G,1G would sometimes be observed. But they are never observed.

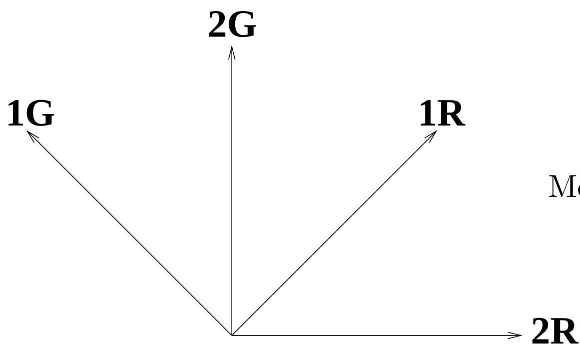
(2) It follows from (1) that in any 22 run in which both detectors flash G (as sometimes happens in 22 runs) each particle must require a type 1 detector to flash R. Therefore if the coin tosses had instead resulted in that particular run being a 11 run, both detectors would have flashed R.

(3) But 1R,1R is never observed

This is Hardy's paradox. It arises directly from the data. Something is wrong with the above reasoning. There is no general agreement about where the error lies. Note that the role of the quantum mechanical formalism is merely to establish that such data is possible, if quantum mechanics is correct in its quantitiative predictions. The paradox is not in the quantum formalism but in the data it gives rise to.

We conclude by showing that quantum mechanics can indeed give rise to such data.

How it works: Devices measure linear polarization of photons



Mode **2** measures 0° (**R**), 90° (**G**) polarization.

Mode **1** measures 45° (**R**), 135° (**G**) polarization.

One photon polarization states: $|2R\rangle, |2G\rangle, |1R\rangle, |1G\rangle$.

$$\langle 2R|2G\rangle = 0,\ \langle 1R|1G\rangle = 0,\ \langle 1R|2R\rangle = \tfrac{1}{\sqrt{2}}.$$

Two photon polarization states: $|1R, 2G\rangle = |1R\rangle|2G\rangle$, etc.

$$\langle 1R, 1R|2R, 2R\rangle = \langle 1R|2R\rangle\langle 1R|2R\rangle = \tfrac{1}{2}.$$

Data produced by pairs of photons in the Hardy polarization state:

$$|\Psi\rangle = c\big(|2R, 2R\rangle - \tfrac{1}{2}|1R, 1R\rangle\big)$$

$$\langle 1R, 1R|\Psi\rangle = c\big(\langle 1R, 1R|2R, 2R\rangle - \tfrac{1}{2}\big) = 0 \;\Rightarrow\; p(1R, 1R) = 0.$$

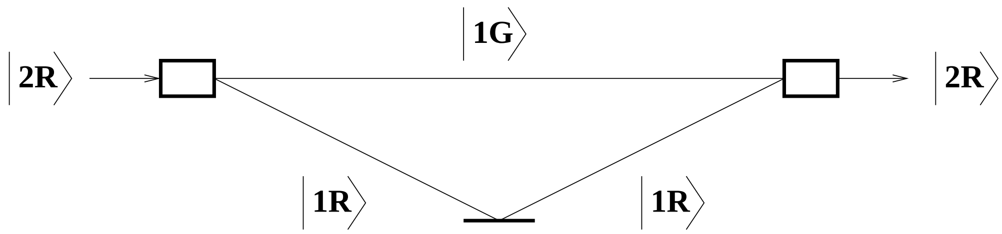
$$\langle 1G, 2G|\Psi\rangle = 0;\quad \langle 2G, 1G|\Psi\rangle = 0 \;\Rightarrow\; p(1G, 2G) = p(2G, 1G) = 0.$$

$$\langle 2G, 2G|\Psi\rangle = c\big((0 - \tfrac{1}{2}\cdot\tfrac{1}{2}\big) = -\tfrac{c}{4} \;\Rightarrow\; p(2G, 2G) = \tfrac{c^2}{16}.$$

$$1 = \langle\Psi|\Psi\rangle = c^2\big((1 + \tfrac{1}{4} - 2\cdot\tfrac{1}{2}\cdot\tfrac{1}{2})\big) = \tfrac{3}{4}c^2 \;\Rightarrow\; p(2G, 2G) = \tfrac{1}{12}.$$

How to make a pair of photons whose polarization state is $|\Psi\rangle$.

A polarization interferometer:



Two such polarization interferometers and a nonlinear gadget: