

Assignment 9

This is the second part of an exercise that examines the stability of a spin-1/2 particle in a magnetic trap. Here you will first identify the leading spin-flip perturbation and finish by calculating the trapping lifetime using the Golden Rule. Refer to assignment 8 for definitions and notation. This assignment is due on Halloween.

1. (a) The first step in identifying the spin-flip perturbation is to evaluate the effective vector potential at the center of the trap. Show that

$$\mathbf{A}|_{x=y=z=0} = -i \frac{\alpha}{2B_0} (\sigma_y \hat{\mathbf{x}} + \sigma_x \hat{\mathbf{y}}).$$

- (b) Collect together all the terms in the transformed Hamiltonian H that involve \mathbf{A} and, using the limiting form from (a), extract the perturbation that flips the spin (from $\sigma_z = -1$ to $\sigma_z = +1$):

$$H_{\text{flip}} = i \frac{\hbar^2 \alpha}{2MB_0} (\sigma_y \frac{\partial}{\partial x} + \sigma_x \frac{\partial}{\partial y}).$$

Is H_{flip} Hermitian?

2. As in assignment 8, set $\alpha = \sqrt{3B_0\beta/2}$ so that the trapping potential is isotropic. Use the Golden Rule with H_{flip} to calculate the lifetime τ of the $\sigma_z = -1$ ground state you found in assignment 8. Here are some things to remember:

- The lifetime τ is the reciprocal of the Golden Rule transition rate Γ summed over all (energy conserving) final states.
- Make the approximation $-\mu B \approx -\mu B_0$ for the final spin-up state. As in the p-state decay calculation in lecture, this neglects the downward curvature of the potential in the open system.
- Keep track of the small and large energies in this decay. The zero point energy of the trapped spin-down particle in its ground state, $\hbar\omega_s/2$, is much smaller than the energy change $\hbar\omega_f = 2\mu B_0$ associated with flipping the spin. For example, the kinetic energy of the final state can be approximated by $\hbar\omega_f$. As in lecture, the subscripts denote “slow” and “fast”.
- Express your final answer for τ in terms of the frequencies ω_s and ω_f , rather than the parameters β and B_0 . How does τ depend on the ratio ω_s/ω_f ?
- The transition matrix element depends on the momentum direction of the final state. **What is the angular distribution of the particles emitted from the trap?**