Assignment 5

In this assignment you have a chance to apply perturbation theory to the quasi-1D hydrogenic states of an electron bound to the flat surface of liquid helium. There are efforts underway to exploit these states in the design of a quantum computer. This assignment is due Monday, October 1, in lecture.

1. The first order of business is to understand the qubits of the proposed quantum computer. In this scheme, electrons are electrostatically bound to the surface of a thick helium film.

   (a) Make a sketch of the electric field lines of a negative point charge (electron) suspended above a uniform dielectric medium (liquid helium) in the region of space $z < 0$. There are no free charges in the dielectric medium, only induced surface charges. **Without actually doing the calculation**, convince yourself[1] that the source of an effective attraction of the electron to the surface of the dielectric could be an image charge, resulting in a potential of the form $V(z) = -\lambda\, e^2/z$. A detailed calculation gives $\lambda = (\epsilon - 1)/4(\epsilon + 1)$, where $\epsilon = 1.057$ is the dielectric constant of liquid helium.

   (b) Find the ground state and first excited state wavefunctions and energies of an electron in the above potential. This is a one dimensional problem, since the electron is in a zero-momentum plane wave state with respect to the transverse coordinates. As in the electron bubble of assignment 4, **the electron wavefunction vanishes at the helium surface**. Describe the relationship between the wavefunctions of this 1D problem and the wavefunctions of the 3D hydrogen atom (specify n and l values). What is the effective Bohr radius (in Å) for electrons on helium?

   The ground and first excited states are the basis states $|0\rangle$ and $|1\rangle$ of a qubit in the proposed quantum computer.

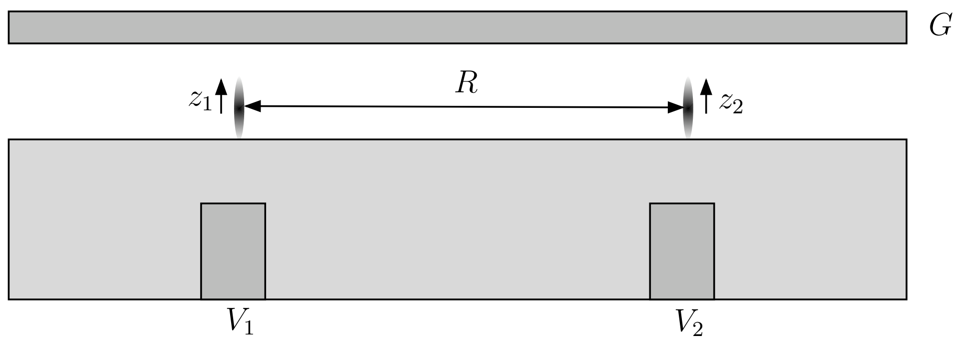
2. By applying an electric field perpendicular to the helium surface, the energies of the basis states of a qubit can be finely tuned.

   (a) Find the frequency ν (in Hz) of the transition between the $|0\rangle$ and $|1\rangle$ states in the absence of an electric field.

   (b) Use perturbation theory to find the shift in frequency per field strength, $\Delta\nu/E_\perp$ (in Hz per Volts/cm).

[1] Take a peek into your favorite electrodynamics textbook.

3. Below is a cartoon of the quantum computer layout proposed by Platzman and Dykman:



Shown are two qubits: electrons hovering over the surface of helium and laterally localized by two gate electrode posts below the helium surface. The transition frequencies $\Delta\nu_1$ and $\Delta\nu_2$ can be precisely tuned to the same value by adjusting the potential difference between the ground plate (G) and the posts (V_1 and V_2)[2].

The two qubits interact (to carry out a quantum logic operation) through the weak dipole-dipole coupling of their electrons' charge distributions:

$$H_{\text{int}} = \tilde{\lambda}\frac{e^2}{R^3}(z_1 - z_0)(z_2 - z_0)\ .$$

As shown in the diagram, R is the separation of the electrons, and z_1, z_2 are the vertical displacements of the electrons relative to the origin of the multipole expansion, z_0. The actual value of z_0 is not needed for this exercise (a natural definition is the vanishing of the dipole moment in the ground state: $\langle 0|(z_i - z_0)|0\rangle = 0$). The ϵ-dependent number $\tilde{\lambda}$ is very close to 1 for the weakly dielectric environment of the two dipoles.

(a) For perfectly tuned gate potentials, the two excited states $|1\rangle|0\rangle$ and $|0\rangle|1\rangle$ of the two-qubit system are exactly degenerate. Evaluate the matrix elements of H_{int} in this basis of degenerate states.

(b) For the qubit separation $R = 1$ micron, calculate the oscillation frequency of the weakly interacting two qubit system. What time is required for the state $|1\rangle|0\rangle$ to evolve into $|0\rangle|1\rangle$?

[2] This is to compensate for frequency shifts caused by small differences in the trapping fields due to imperfections in the nanofabricated posts.