Diagnostic Quiz Feedback

Going over your answers has been an informative experience. Below are some brief remarks and recommendations that may or may not apply to the answer you gave to the question.

1. What textbook was used in your most advanced course on quantum mechanics?

   I used Schiff, 3rd edition, as an undergraduate.

2. What is your favorite quantum mechanics text?

   I completely switched over to the path integral approach as a graduate student, and therefore was fond of the book by Feynman and Hibbs. Today I take a more balanced view, and as a result have no great attachment to any book.

3. Can you recall the formula for the Bohr radius, as expressed in terms of fundamental physical constants? If not, how would you derive the formula?

   Many of you used Bohr's trick of computing the angular momentum for a circular orbit and setting it's value equal to $n\hbar$. The orbit with $n = 1$ gives the correct ground state energy, which is fine until you remember the ground state has angular momentum zero. The correct semi-classical orbit to use for the ground state is the orbit where the electron falls radially inward and passes through the nucleus (and oscillates in this manner). A better and more direct approach is to apply scaling ideas to the terms in the Hamiltonian (first lecture).

4. The combination $e^2/\hbar c$ is said to be a dimensionless constant — a pure number. Do you recall its order of magnitude? What happened to the $(\mathrm{Coulomb})^2$ units in the numerator?

   This was answered in the first lecture. To remember its value, recall "shaken, not stirred."

5. What is the order of magnitude of v/c, where v is the speed of the electron in a ground state hydrogen atom?

   This was answered in the second lecture.

6. What, in your expert opinion, is the fundamental explanation of the fact that it is not possible to walk through walls?

   This will be dealt with in the second homework assignment. Quantum mechanical tunneling is **not** the issue here.

7. A hydrogen atom is in its ground state. Within the Born-Oppenheimer approximation, where the proton may be considered fixed at the origin, sketch the wave function of the electron on a line that passes through the origin (e.g. along the positive and negative x-axis):

   This was shown in the first lecture. The cusp at the origin is real — not a mathematical artifact.

8. A hydrogenic atom with nuclear charge Z is in its ground state. The nucleus then decays spontaneously via an energetic beta decay process with the result that the final nucleus has charge $Z - 1$. Is the electron after the decay, assumed to be essentially instantaneous, still in the ground state of the atom? Would you describe the state of the electron as "stationary"?

   Think of the original wavefunction as the initial condition for a time-dependent Schrödinger equation. The original wavefunction is not stationary with respect to the new equation because the potential has been changed. Applying a new equation to an old wavefunction (in the sense of an initial condition) is justified here because the time scale of the transit of the positron across the atom (beta decay) is much faster than the electronic time scale. You learned about the electronic time scale in the second lecture.

9. Two particles interact via a force whose range is less than d (the pair potential $V(r)$ is zero for separations $r > d$). Can the particles be in a bound state if the expectation value of their separation equals $10\,d$?

   This is possible. As an extreme example, consider the limit of a delta-function potential (assignment 1).

10. What, in your expert opinion, is the fundamental reason for the bent shape of the H_2O molecule?

    Stay in the course to find out. The explanation is a surprisingly general consequence of the Pauli principle.

11. A particle is in an approximate plane wave state in the vicinity of the origin. Does this particle have a definite value of angular momentum about the origin, and if so, what is this value?

    In this and the following problem it's important to understand the difference between "definite value" and "expected value." A plane wave is actually a superposition of infinitely many states with different, definite values of angular momentum — the so-called partial wave expansion.

12. Write down, in your favorite notation, a state of a photon whose spin angular momentum expectation value is $\hbar/2$ — or argue that this is impossible.

    Here's such a state: $\frac{\sqrt{3}}{2}|+\rangle + \frac{1}{2}|-\rangle$.

    The basis states correspond to right- and left-handed circular polarization, with $L_z = \pm\hbar$. Note that the answer is not unique.

13. What, in your expert opinion, is the quantum mechanical principle that determines the maximum mass of a compact star, beyond which it becomes unstable to collapse into a neutron star?

    Pauli again. This is my all-time favorite principle.

14. Evaluate the following integral involving the product of two Dirac delta functions:

$$\int_{-\infty}^{\infty} \delta(x-z)\delta(y-z)\,dz = \delta(x-y)$$

    Those of you who wrote

$$\begin{cases} 1 & \text{for} \quad x = y \\ 0 & \text{for} \quad x \neq y \end{cases}$$

    need to review Dirac delta functions.

15. Simplify the commutator $[p^2, x]$:

    Okay, it's equal to $-2i\hbar p$, but I was mostly looking to see if you seemed to have some idea what you were doing and got the units right.

16. Which of the following matrices, if any, are unitary:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \qquad \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

    Too many of you were computing $U^\dagger U$ and checking if the result was the identity matrix. The rows (and columns) of a unitary matrix are orthonormal in the Hermitian sense: components in any one row satisfy $|\alpha|^2 + |\beta|^2 = 1$, and for different rows, $\alpha_1\alpha_2^* + \beta_1\beta_2^* = 0$. I don't think I'm unusual in being able to do this (for the matrices above) in my head.

17. What word describes the matrix $V = e^{iT}$, given that T is a Hermitian matrix? What does the previous equation remind you of?

    V is unitary and the whole equation is a mnemonic for spelling my name.

18. Evaluate the following derivative, where A and B are $n \times n$ matrices:

$$\frac{d}{dt}\left(e^{A+Bt}\right) =$$

    You saw this at the start of the first lecture. I remember this from one of my undergraduate quantum exams.

19. Let $|lm\rangle$ denote a state with angular momentum quantum numbers l and m, and $|\mathbf{r}\rangle$ a state of definite position $\mathbf{r}$ on the unit sphere. Evaluate the following and express your answers in terms of the standard spherical coordinates of $\mathbf{r}$:

$$\begin{aligned} \langle 00|\mathbf{r}\rangle &= \\ \langle 11|\mathbf{r}\rangle &= \\ \langle 10|\mathbf{r}\rangle &= \end{aligned}$$

Congratulations to those of you who understood this was asking for the three most widely used spherical harmonic functions (Y_{1m}). It's a good idea to learn these **now**, and save yourself the trouble of looking them up dozens of times in the next semester.

20. Use the wavefunction $\psi(x) = \exp(-\alpha|x|)$ with parameter $\alpha > 0$ to find a variational bound on the ground state energy of the (dimensionless) harmonic oscillator Hamiltonian:

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 \ .$$

You really should have learned the variational principle by now. Several of you obtained a negative number for the expectation value of the kinetic energy. This should have raised a flag that something was seriously wrong.