Diagnostic Quiz

Please provide short answers to all of the questions and do **not** sign your name to this document. Your responses will help the instructor assess the level of your experience with quantum mechanics. Frank responses are more useful than cautious ones.

1. What textbook was used in your most advanced course on quantum mechanics?

2. What is your favorite quantum mechanics text?

3. Can you recall the formula for the Bohr radius, as expressed in terms of fundamental physical constants? If not, how would you derive the formula?

4. The combination $e^2/\hbar c$ is said to be a dimensionless constant — a pure number. Do you recall its order of magnitude? What happened to the $(\mathrm{Coulomb})^2$ units in the numerator?

5. What is the order of magnitude of v/c, where v is the speed of the electron in a ground state hydrogen atom?

6. What, in your expert opinion, is the fundamental explanation of the fact that it is not possible to walk through walls?

7. A hydrogen atom is in its ground state. Within the Born-Oppenheimer approximation, where the proton may be considered fixed at the origin, sketch the wave function of the electron on a line that passes through the origin (e.g. along the positive and negative x-axis):

8. A hydrogenic atom with nuclear charge Z is in its ground state. The nucleus then decays spontaneously via an energetic beta decay process with the result that the final nucleus has charge $Z - 1$. Is the electron after the decay, assumed to be essentially instantaneous, still in the ground state of the atom? Would you describe the state of the electron as "stationary"?

9. Two particles interact via a force whose range is less than d (the pair potential $V(r)$ is zero for separations $r > d$). Can the particles be in a bound state if the expectation value of their separation equals $10\,d$?

10. What, in your expert opinion, is the fundamental reason for the bent shape of the H_2O molecule?

11. A particle is in an approximate plane wave state in the vicinity of the origin. Does this particle have a definite value of angular momentum about the origin, and if so, what is this value?

12. Write down, in your favorite notation, a state of a photon whose spin angular momentum expectation value is $\hbar/2$ — or argue that this is impossible.

13. What, in your expert opinion, is the quantum mechanical principle that determines the maximum mass of a compact star, beyond which it becomes unstable to collapse into a neutron star?

14. Evaluate the following integral involving the product of two Dirac delta functions:

$$\int_{-\infty}^{\infty} \delta(x-z)\delta(y-z)\,dz =$$

15. Simplify the commutator $[p^2, x]$:

16. Which of the following matrices, if any, are unitary:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \qquad \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

17. What word describes the matrix $V = e^{i\,T}$, given that T is a Hermitian matrix? What does the previous equation remind you of?

18. Evaluate the following derivative, where A and B are $n \times n$ matrices:

$$\frac{d}{dt}\left(e^{A+Bt}\right) =$$

19. Let $|lm\rangle$ denote a state with angular momentum quantum numbers l and m, and $|\mathbf{r}\rangle$ a state of definite position $\mathbf{r}$ on the unit sphere. Evaluate the following and express your answers in terms of the standard spherical coordinates of $\mathbf{r}$:

$$\begin{aligned} \langle 00|\mathbf{r}\rangle &= \\ \langle 11|\mathbf{r}\rangle &= \\ \langle 10|\mathbf{r}\rangle &= \end{aligned}$$

20. Use the wavefunction $\psi(x) = \exp(-\alpha|x|)$ with parameter $\alpha > 0$ to find a variational bound on the ground state energy of the (dimensionless) harmonic oscillator Hamiltonian:

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 \ .$$

21. What quantum mechanical phenomena would you like to learn more about?

22. What topics, if covered, do you feel would be a waste of your time?

23. What other physics courses are you considering this semester?